Chapter 2

Sets

2.1 Set

A set is an unordered collection of objects, called elements or members of the set. A set is said to contain its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

There are several ways to describe a set. One way is to list all the members of a set, when this is possible. We use a notation where all members of the set are listed between braces. For example, the notation a, b, c, d represents the set with the four elements $a, b, c,$ and d. This way of describing a set is known as the roster method.

Another way to describe a set is to use set builder notation. We characterize all those elements in the set by stating the property or properties they must have to be members. For instance, the set O of all odd positive integers less than 10 can be written as $O = \{x|x$ is an odd positive integer less than $10\}$,

or, specifying the universe as the set of positive integers, as

$O = \{x|x \in Z^+|$ is odd and $x < 10\}$

Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$ if A and B are equal sets

The set A is a subset of B if and only if every element of A is also and element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.

Showing that A is a Subset of B To show that $A \subseteq B$, Show that if x belongs to A then x also belongs to B

Showing that A is Not a Subset of B To show that $A \not\subseteq B$, find a single $x \in A$ such that $x \notin B$.

Showing Two Sets are Equal To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.

Given a set S, the power set of S is the set of all subsets of the set S. The power set of S is denoted by $P(S)$.

Let A and B be the sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ abd $b \in B$. Hence,
$A \times B = \{(a, b) | a \in A \wedge b \in B\}$.

The order of elements in a collection is often important. Because sets are unordered, a different structure is needed to represent ordered collections. This is provided by ordered $n-$tuples.

The Cartesian product of the sets $A_1, A_2, ..., A_n$ denoted by $A_1 \times A_2 \times A_3 \times ... \times A_n$ is the set of ordered $n-$tuples $(a_1, a_2, ..., a_n)$, where a_i belongs to A_i for $i = 1, 2, ..., n$. In other words,
$A_1, A_2, ..., A_n = \{(a_1, a_2, ..., a_n) | a_i \in A_i \text{ for } i = 1, 2, ..., n\}$.

Let A and B be sets. The union of the sets A and B, denoted by $A \cup B$, is the set containing those elements that are either in A and B or in both.

Let A and B be sets. The intersection of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

Two sets are called disjoint if their intersection is the empty set.

Let A and B be sets. The difference of the sets A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.

Let U be the universal set. The complement of the set A, denoted by $\overline{A}$, is the complement of A with respect to U. Therefore, the complement of the set A is $U - A$.
$\overline{A} = \{x \in U | x \notin A\}$.

Set Identities	
$A \cap U = A$ $A \cup \phi = A$	Identity laws
$A \cup U = U$ $A \cap \phi = \phi$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation Law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \phi$	Complement laws

Set identities can also be proved using **membership tables**. We consider each combination of sets that an element can belong to and verify that elements in the same combinations of sets belong to both sets in the identity. To indicate that an element is in a set, a 1 is used; to indicate that an element is not in the set, a 0 is used (The reader should note the similarities between membership tables and truth tables).

Use a membership table to show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Solution: The membership table for each combinations of sets is shown in Table below. This table has eight rows. Because the columns for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are the same, the identity is valid.

A Membership Table for the Distributive Property							
A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

Computer Representation of sets

There are various ways to represent sets using a computer. One method is to store the elements of the set in an unordered fashion. However, if this is done, the operations of computing the union, intersection, or difference of two sets would be time-consuming, because each of these operations would require a large amount of searching for elements.

Assume that the universal set U is finite (and of reasonable size so that the number of elements of U is not larger than the memory size of the computer being used). First, specify an arbitrary ordering of the elements of U, for instance $a_1, a_2, ..., a_n$. Represent a subset A of U with the bit string of length n, where the i^{th} bit in this string is 1 if a_i belongs to A and is 0 if does not belong to A.

Example - Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and this ordering of elements of U has the elements in increasing order; that is $a_i = i$. What bit strings represent the subset of all odd integers not exceeding 5 in U?
Ans: 10 1010 1010, 01 0101 0101, 11 1110 0000.