

imum number of coins?

18. Six Jobs with their Job_ID, Deadlines, Profits and Arrival-times respectively are represented in tuples as follows:

(a,2,200,0), (b,1,19,0), (c,2,27,1), (d,1,25,1), (e,3,15,0),(f,1,80,0)

, where Deadline is defined as number of wait-time units before the job expires while Arrival time is the time-instance when the process joined the Job-queue. Assuming that a job gets finished as soon as it is dispatched to be executed.

What should be the order of these jobs in order to maximize the overall profit of the sequence. Also write the maximum achievable profit of the sequence obtained.

9 Graphs

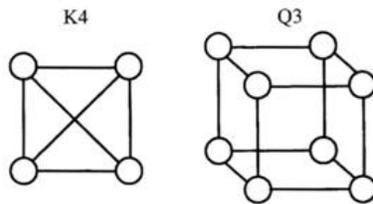
- Let G be a complete undirected graph on 8 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to
 - 150
 - 90
 - 140
 - 210
 - 70
- Which of the following statements is/are TRUE for undirected graphs?

P: Number of even degree vertices is even.
Q: Sum of degrees of all odd-degree vertices is even.
R: Sum of degrees of all vertices is even.

 - P only
 - Q only
 - R only
 - Both P and Q
 - Both Q and R
 - All of the above
 - None of the above
- Consider an undirected random graph of ten vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?
 - $\frac{1}{8}$
 - $\frac{1}{10}$

- (C) 10
- (D) 15
- (E) 120
- (F) 60

4. Which of the following statements about these two graphs is true:



- (A) K_4 is planar while Q_3 is not
- (B) Both K_4 and Q_3 are planar
- (C) Q_3 is planar while K_4 is not
- (D) Neither K_4 nor Q_3 are planar

5. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

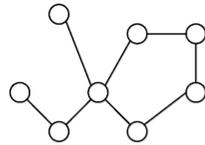
- I. 7, 6, 5, 4, 4, 3, 2, 1
- II. 6, 6, 6, 6, 3, 3, 2, 2
- III. 7, 6, 6, 4, 4, 3, 2, 2
- IV. 8, 7, 7, 6, 4, 2, 1, 1

- (A) I and II
- (B) III and IV
- (C) IV only
- (D) II and IV

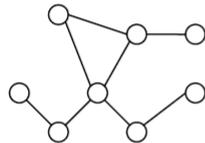
6. Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of unbounded faces in any embedding of G on the plane is equal to

- (A) 1
- (B) 4
- (C) 3
- (D) 6
- (E) 5

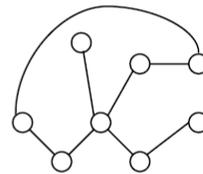
7. Which of the following four graphs is isomorphic to this graph given below:



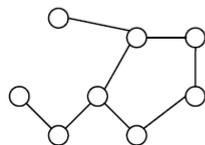
(A)



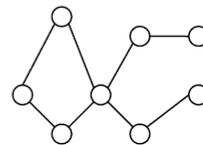
(B)



(C)



(D)



- (A) A
- (B) B
- (C) C
- (D) D

8. What is the chromatic number of an n -vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 4$.

- (A) 2
- (B) 3
- (C) $n-1$
- (D) n

9. Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?

- (A) No two vertices have the same degree.
- (B) At least two vertices have the same degree.
- (C) At least three vertices have the same degree.
- (D) All vertices have the same degree

10. The line graph $L(G)$ of a simple graph G is defined as follows:

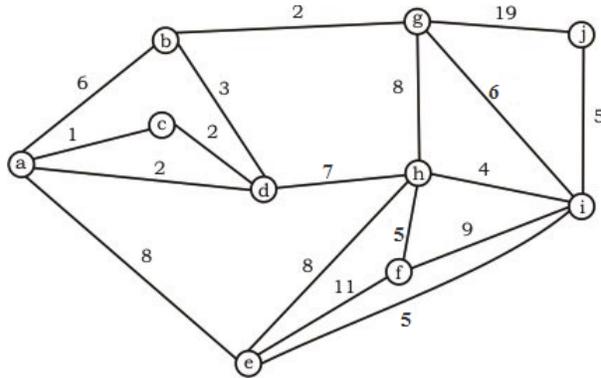
There is exactly one vertex $v(e)$ in $L(G)$ for each edge e in G . · For any two edges e and e' in G , $L(G)$ has an edge between $v(e)$ and $v(e')$, if and only if e and e' are incident with the same vertex in G . Which of the following statements is/are TRUE?

- (P) The line graph of a cycle is a cycle.
 - (Q) The line graph of a clique is a clique.
 - (R) The line graph of a planar graph is planar.
 - (S) The line graph of a tree is a tree.
- (A) P only
(B) P and R only
(C) R only
(D) P, Q and S only
11. Let G be the non-planar complete graph with the minimum possible number of edges. Then G has
- (A) 9 edges and 5 vertices
 - (B) 9 edges and 6 vertices
 - (C) 10 edges and 5 vertices
 - (D) 10 edges and 6 vertices
12. Which of the following graphs has an Eulerian circuit?
- (A) Any k -regular graph where k is an even number.
 - (B) A complete graph on 90 vertices
 - (C) The complement of a cycle on 25 vertices
 - (D) None of the above
13. Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $(i, j): 1 \leq i \leq 12, 1 \leq j \leq 12$. There is an edge between (a, b) and (c, d) if $|a - c| \leq 1$ and $|b - d| \leq 1$. The number of edges in this graph is
- (A) 500
 - (B) 502
 - (C) 506
 - (D) 510
14. The maximum number of edges in a tripartite graph with 13 vertices is
- (A) 54
 - (B) 58
 - (C) 80
 - (D) 56
 - (E) 64

15. A cycle on n vertices is isomorphic to its complement. The value of n is
(A) 2
(B) 4
(C) 6
(D) 5
16. Maximum number of edges in a n - node undirected graph without self loops is
(A) n^2
(B) $\frac{n(n-1)}{2}$
(C) $n - 1$
(D) $\frac{(n+1)n}{2}$
17. Which of the following statements is true for every planar graph on n vertices?
(A) The graph is connected
(B) The graph is Eulerian
(C) The graph has a vertex-cover of size at most $\frac{3n}{4}$
(D) The graph has an independent set of size at least $\frac{n}{3}$
18. G is a graph on n vertices and $2n - 2$ edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G ?
(A) For every subset of k vertices, the induced subgraph has at most $2k-2$ edges
(B) The minimum cut in G has at least two edges
(C) There are two edge-disjoint paths between every pair of vertices
(D) There are two vertex-disjoint paths between every pair of vertices
19. In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is True?
(A) A tree has no bridge
(B) A bridge cannot be part of a simple cycle
(C) Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)
(D) A graph with bridges cannot have a cycle
20. If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a
(A) Hamiltonian cycle
(B) grid
(C) hypercube

(D) tree

21. What is the weight of a minimum spanning tree of the following graph ?



- (A) 33
 (B) 31
 (C) 38
 (D) 37

22. If G is a forest with n vertices and k connected components, how many edges does G have?

- (A) $\lfloor \frac{n}{k} \rfloor$
 (B) $\lceil \frac{n}{k} \rceil$
 (C) $n-k$
 (D) $n-k+1$

23. Consider this data for following three questions:

The 2^n vertices of a graph G corresponds to all subsets of a set of size n , for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements. The number of vertices of degree zero in G is:

- (A) 1
 (B) n
 (C) $n+1$
 (D) 2^n

24. In above question what is the number of connected components in G ?

- (A) n
 (B) $n+2$
 (C) $2^{\frac{n}{2}}$

- (D) $\frac{2^n}{n}$
25. In above question, the maximum degree of a vertex in G is:
(A) $\binom{\frac{n}{2}}{2} \cdot 2^{\frac{n}{2}}$
(B) 2^{n-2}
(C) $2^{n-3} * 3$
(D) 2^{n-1}
26. The minimum number of colours required to colour the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same colour is
(A) 2
(B) 3
(C) 4
(D) $n - 2\lfloor \frac{n}{2} \rfloor + 2$
27. Consider a full binary tree with n internal nodes, internal path length i, and external path length e. The internal path length of a full binary tree is the sum, taken over all nodes of the tree, of the depth of each node. Similarly, the external path length is the sum, taken over all leaves of the tree, of the depth of each leaf. Which of the following is correct for the full binary tree?
(A) $e = i+n$
(B) $e = i+2n$
(C) $e = 2i+n$
(D) $e = 2^n + i$
28. Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between
(A) k and n
(B) k - 1 and k + 1
(C) k - 1 and n - 1
(D) k + 1 and n - k
29. Let G be an undirected connected graph with distinct edge weight. Let E_{max} be the edge with maximum weight and E_{min} the edge with minimum weight. Which of the following statements is false?
(A) Every minimum spanning tree of G must contain E_{min} .
(B) If E_{max} is in minimum spanning tree, then its removal must disconnect G.
(C) No minimum spanning tree contains E_{max}

- (D) G has a unique minimum spanning tree.
30. How many perfect matchings are there in a complete graph of 9 vertices ?
(A) 84
(B) 384
(C) 784
(D) 984
(E) None of the above
31. Let $G = (V, E)$ be any connected undirected edge-weighted graph. The weights of the edges in E are positive. Consider the following statements:
(1) The path between a pair of vertices in a minimum spanning tree of an undirected graph is necessarily the shortest (minimum weight) path.
(2) Minimum Spanning Tree of G is always unique and shortest path between a pair of vertices may not be unique.
Which of the above statements is/are necessarily true?
(A) (1) only
(B) (2) only
(C) both (1) and (2)
(D) neither (1) nor (2)
32. An undirected graph $G(V, E)$ contains n ($n > 2$) nodes named v_1, v_2, \dots, v_n . Two nodes v_i and v_j are connected if and only if $0 < |i - j| \leq 2$. Each edge (v_i, v_j) is assigned a weight $i + j$. The cost of the minimum spanning tree of such a graph with 20 nodes is:
(A) 880
(B) 491
(C) 381
(D) 478
(E) 378
33. Consider a weighted undirected graph with positive edge weights and let uv be an edge in the graph. It is known that the shortest path from the source vertex s to u has weight 79 and the shortest path from s to v has weight 55. Which one of the following statements is always true?
(A) $\text{weight}(u, v) < -24$
(B) $\text{weight}(u, v) \leq 24$
(C) $\text{weight}(u, v) > -24$
(D) $\text{weight}(u, v) \geq 24$
(E) $\text{weight}(u, v) \geq -24$
(F) There exists some edge in G with negative weight.

34. Consider the following statements
- (I) Let T be a binary search tree with 4 height. The minimum and maximum possible nodes of T are 5 and 15 respectively.
 - (II) In a binary tree, the number of internal nodes of degree 2 is 6, and the number of internal nodes of degree 1 is 8. The number of leaf nodes in the binary tree is 15.
- Which of the following statement(s) is/are correct?
- (A) Only (I)
 - (B) Only (II)
 - (C) Both (I) and (II)
 - (D) Neither (I) nor (II)
35. The number of possible min-heaps containing each value from 1, 2, 3, 4, 5, 6, 7 exactly once is?
- (A) 80
 - (B) 100
 - (C) 20
 - (D) 50
 - (E) 48
36. The line graph $L(G)$ of a simple graph G is defined as follows:
There is exactly one vertex $v(e)$ in $L(G)$ for each edge e in G .
For any two edges e and e' in G , $L(G)$ has an edge between $v(e)$ and $v(e')$, if and only if e and e' are incident with the same vertex in G .
Which of following option is not correct about "Line Graph"?
- (A) A line graph has an articulation point if and only if the underlying graph has a bridge for which neither endpoint has degree one
 - (B) For a graph G with n vertices and m edges, the number of vertices of the line graph $L(G)$ is m , and the number of edges of $L(G)$ is half the sum of the squares of the degrees of the vertices in G , m .
 - (C) If a graph G has an Euler cycle, that is, if G is connected and has an even number of edges at each vertex, then the line graph of G is Hamiltonian.
 - (D) None of these
37. Let G be a graph with $100!$ vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, ..., 100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v . Let y denote the degree of a vertex in G , and z denote the number of connected components in G . Then $10y + z = ?$
- (A) 99
 - (B) 100

- (C) 109
- (D) 119
- (E) 991
- (F) None of these

10 Hashing

1. Which of the following statement(s) is TRUE?
 - (I) A hash function takes a message of arbitrary length and generates a fixed length code.
 - (II) A hash function takes a message of fixed length and generates a code of variable length.
 - (III) A hash function may give the same hash value for distinct messages.
 - (IV) Searching a key in a hash table takes linear amount of time at worst case.
 - (A) (I) and (IV) only
 - (B) (II) and (III) only
 - (C) (I) and (III) only
 - (D) (II) and (IV) only
 - (E) (IV) only
2. A hash table of length 10 uses open addressing with hash function $h(k)=k \bmod 10$, and linear probing.

0	29
1	11
2	82
3	43
4	34
5	
6	96
7	117
8	
9	289

After inserting 8 values into an empty hash table, the table is as shown below. Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

- (A) 29, 11, 82, 43, 34, 117, 289, 96
- (B) 11, 82, 43, 34, 29, 117, 289, 96