Chapter 5

Sets

1. Let A and B be sets. Show that
   (a) $(A \cap B) \subseteq A$
   (b) $A \subseteq (A \cup B)$
   (c) $A - B \subseteq A$
   (d) $A \cap (B - A) = \phi$
   (e) $A \cup (B - A) = A \cup B$
2. Show that if A, B and C are sets then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$
3. Let A, B and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.
4. Can you conclude that A = B if A, B and C are sets such that
   (a) $A \cup C = B \cup C$
   (b) $A \cap C = B \cap C$
   (c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$
5. Let A and B be subsets of a universal set U. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.

   The symmetric difference of A and B, denoted by $A \oplus B$ is the set containing those elements in either A or B, but not in both A and B.
6. Show that $A \oplus B = (A \cup B) - (A \cap B)$.
7. Show that $A \oplus B = (A - B) \cup (B - A)$.
8. Show that if A is a subset of the universal set U. then
   (a) $A \oplus A = \phi$
   (b) $A \oplus \phi = A$

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   (c) $A \oplus U = \overline{A}$
   (d) $A \oplus \overline{A} = U$

9. Show that if A and B are sets, then
   (a) $A \oplus B = B \oplus A$
   (b) $(A \oplus B) \oplus B = A$

10. What can you say about the sets A and B if $A \oplus B = A$?

11. Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$ Find-
   (a) $\bigcup_{i=1}^{n} A_i$
   (b) $\bigcap_{i=1}^{n} A_i$

12. Let $A_i = \{\ldots, -2, -1, 0, 1, \ldots, i\}$ Find-
   (a) $\bigcup_{i=1}^{n} A_i$
   (b) $\bigcap_{i=1}^{n} A_i$

13. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i, $A_i = \{i, i+1, i+2, \ldots\}$.

14. The successor of the set A is the set $A \cup \{A\}$. Find the successors of the following sets.
   (a) $\{1, 2, 3\}$
   (b) ϕ
   (c) $\{\phi\}$
   (d) $\{\phi, \{\phi\}\}$

15. How many elements does the successor of a set with n elements have ?

16. Let P and Q be multi-sets. The **union** of the multi-sets P and Q is the multi-set where the multiplicity of an element is the maximum of its multiplicities in P and Q. The **intersection** of P and Q is the multi-set where the multiplicity of an element is the minimum of its multiplicities in P and Q. The **difference** of P and Q is the multi-set where the multiplicity of an element is the multiplicity of the element in P less its multiplicity in Q unless this difference is negative, in which case the multiplicity is 0. The **sum** of P and Q is the multi-set where the multiplicity of an element is the sum of multiplicities in P and Q. The union, intersection and difference of P and Q are denoted by $P \cup Q, P \cap Q$ and $P - Q$, respectively (where these operations should not be confused with the analogous operations for sets). The sum of P and Q is denoted by $P + Q$.

    Let A and B be the multi-sets $\{3.a, 2.b, 1.c\}$ and $\{2.a, 3.b, 4.d\}$ respectively. Find

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(a) $A \cup B$
(b) $A \cap B$
(c) $A - B$
(d) $B - A$
(e) $A + B$

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