Chapter 1

Propositional Logic

1. Show that these conditional statements is a tautology

   (a) $[\neg p \wedge (p \vee q)] \rightarrow q$

   (b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

   (c) $[p \wedge (p \rightarrow q)] \rightarrow q$

   (d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

2. Show that given $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent.

3. Show that given $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

4. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

5. Show that given $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ are logically equivalent.

6. Show that given $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

7. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

8. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent

9. Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.

10. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

11. Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.

12. Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

The **dual** of a compound proposition that contains only the logical operators $\vee, \wedge$ and $\neg$ is the compound proposition obtained by replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$, each **T** by **F**, and each **F** by **T**. The dual of s is denoted by s^*.

13. Find the dual of each of these compound propositions.
    (a) $p \vee \neg q$
    (b) $p \wedge (q \vee (r \wedge T))$
    (c) $(p \wedge \neg q) \vee (q \wedge F)$
14. When does $S^* = S$, where S is a compound proposition? (E)
15. Show that $S^{**} = S$ when S is a compound proposition? (D)
16. Why are the duals of two equivalent compound propositions also equivalent, where these compound propositions contain only the operators $\wedge, \vee$ and $\neg$? (H)
17. Which of the following propositional formulas is a tautology?
    (a) $(\neg p \vee r) \rightarrow (p \vee \neg r)$
    (b) $\neg(p \rightarrow (p \wedge q))$
    (c) $r \rightarrow (p \wedge \neg r)$
    (d) $(p \leftrightarrow q) \vee (p \leftrightarrow \neg q)$
18. Which of the following inference system is valid?
    (a) $(p \vee q) \rightarrow r, r \models \neg p$
    (b) $p, \neg p \leftrightarrow q \models \neg q$
    (c) $(p \wedge q) \rightarrow r, \neg r \models p \vee q$
    (d) $p \wedge q \models q \leftrightarrow \neg q$
19. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is a male". If $f(u, w)$ equals $M(v) \wedge \exists x \exists y (P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w))$, The meaning of the expression $F(v, w)$ is
    (a) v is a brother of w
    (b) v is an uncle of w
    (c) v is a grandfather of w
    (d) v is a nephew of w
20. Consider the following inference system:
    $P, \sim P \vee Q, \sim Q \vee R$
    Which of the following is a valid conclusion?
    (a) R
    (b) $\sim R$
    (c) $\sim Q$
    (d) $\sim R \wedge Q$

21. $(P \to A) \vee (P \to B) \vee (P \to C) \vee (P \to D) \vee ... \vee (P \to X) \vee (P \to Y) \vee (P \to Z)$ is equivalent to

    I T

    II F

    III $P \to (A \vee B \vee C \vee D \vee ... \vee X \vee Y \vee Z)$

    (a) I only
    (b) III only
    (c) I & III only
    (d) I, II & III

22. Which of the following is true about the propositional formula
    $[(P \to Q) \wedge (P \to R)] \to [P \to (Q \wedge R)]$

    (a) Not valid
    (b) Valid but not satisfiable
    (c) Not satisfiable and not valid
    (d) Valid and Satisfiable

23. Which of the following propositional formulas represents the sentence, 'He will come on the 8:15 or the 9:15 train; if the former. he will have time to visit us' where
    p means 'He will come on the 8:15'
    q means 'He will come on the 9:15'
    r means 'He will have time to visit us'

    (a) $p \to q \vee r$
    (b) $p \vee q \to r$
    (c) $(p \to q) \wedge (p \vee r)$
    (d) $(p \vee q) \wedge (p \to r)$

24. $S_1 : (P \vee Q)R \Rightarrow (P \to R) \vee (Q \to R)$
    $S_2 : (P \to Q) \vee R \Rightarrow (P \vee R) \to (Q \vee R)$
    Which of the following statements is true?

    (a) Only S_1
    (b) Only S_2
    (c) S_1 and S_2
    (d) Neither S_1 nor S_2

25. Consider the following statement "Nobody loves everybody"
    $L(x, y) = x$ loves y Which of the following statements is true?

    (a) $\forall x \exists y (L(x, y))$

(b) $\exists y \forall x(\neg L(x,y))$
(c) $\forall x \exists y(\neg L(x,y))$
(d) $\exists y \forall x(L(x,y))$

26. A binary operator is defined as follows
$P \Updownarrow Q = \sim P \wedge Q$
Which of the following statement is equivalent to $P \rightarrow Q$
(a) $\sim P \Updownarrow Q$
(b) $\sim (P \Updownarrow Q)$
(c) $\sim (\sim P \Updownarrow Q)$
(d) $\sim (\sim P \Updownarrow \sim Q)$

27. $\neg(P \wedge Q) \vee (P \vee Q)$ is equivalent to
(a) $\neg(P \vee Q) \wedge (P \wedge Q)$
(b) $\neg(\neg(P \vee Q) \wedge (P \wedge Q))$
(c) F
(d) $\neg P \vee \neg Q$

28. S_1 : Every contingency is satisfiable
S_2 : Every satisfiable formula is contingent
(a) Only S_1
(b) Only S_2
(c) $S_1 \& S_2$ both are true
(d) Neither S_1 nor S_2

29. Consider the following statement "Ramu Reserves all the boats".
$R(x,y) = x$ reserves y
$B(y) = y$ is a boat.
I $R(Ramu, y)$
II $x(R(x,y) \wedge x =' ramu')$
(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

30. The formula $\{(p \rightarrow \sim q) \wedge (r \rightarrow q) \wedge r\} \rightarrow p$ is
(a) a tautology
(b) a contingency
(c) a contradiction
(d) None of these