# Chapter 6

# Functions

1. Determine whether f is a function from Z to R if $f(n) = \frac{1}{n^2-4}$.
2. Determine whether f is a function from the set of all bit strings to the set of integers if f(S) is the smallest integer i such that the $i^{th}$ bit of S is 1 and f (S) = 0 when S is the empty string, the string with no bits.
3. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function. (a) The function that assigns to each bit string the number of ones in the string minus the number of zeros in the string
4. Determine whether each of these functions from Z to Z is one-to-one.
   (a) $f(n) = n - 1$
   (b) $f(n) = n^2 + 1$
   (c) $f(n) = n^3$
   (d) $f(n) = \lceil n/2 \rceil$
5. Determine whether $f : Z \times Z \to Z$ is onto if
   (a) $f(m, n) = 2m - n$
   (b) $f(m, n) = |m| - |n|$
   (c) $m^2 - 4$
6. Determine whether each of these functions is a bijection from R to R.
   (a) $f(x) = \frac{x^2+1}{x^2+2}$
   (b) $x^5 + 1$
7. Let $f : R \to R$ and let $f(x) > 0$ for all $x \in R$. Show that f(x) is strictly increasing if and only if the function $g(x) = 1/f(x)$ is strictly decreasing.
8. Suppose that g is a function from A to B and f is a function from B to C.

   (a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.
   (b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

9. If f and $f \circ g$ are onto, does it follow that g is onto? Justify your answer.

10. Show that the function f (x) = ax + b from R to R is invertible, where a and b are constants, with $a \neq 0$, and find the inverse of f.

11. Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that

    (a) $f(S \cup T) = f(S) \cup f(T)$
    (b) $f(S \cap T) \subseteq f(S) \cap f(T)$

12. Let f be a function from the set A to the set B. Let S be a subset of B.We define the inverse image of S to be the subset of A whose elements are precisely all pre-images of all elements of S.We denote the inverse image of S by$f^{-1}(S)$ so $f^{-1}(S) = \{a \in A|f(a) \in S\}$. Let f be a function from A to B. Let S and T be subsets of B. Show that

    (a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
    (b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$