Chapter 2

FOL

1. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

   Friend(x,y) : x is a friend of y
   Perfect(x) : x is perfect

   (a) No one is perfect.
   (b) Not everyone is perfect.
   (c) All your friends are perfect
   (d) At least one of your friends is perfect.
   (e) Everyone is your friend and is perfect.
   (f) Not everybody is your friend or someone is not perfect.

2. Translate these specifications into English where

   F(p) is "Printer p is out of service,"
   B(p) is "Printer p is busy,"
   L(j) is "Print job j is lost,"
   Q(j) is "Print job j is queued."

   (a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$
   (b) $\forall p B(p) \rightarrow \exists j Q(j)$
   (c) $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$
   (d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

3. Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

4. Show that $\exists x(P(x) \vee Q(x))$ and $\exists x P(x) \vee \exists x Q(x)$ are logically equivalent.

5. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty a
   (a) $(\forall x P(x)) \vee A \equiv \forall x(P(x) \vee A)$
   (b) $(\exists x P(x)) \vee A \equiv \exists x(P(x) \vee A)$
   (c) $(\forall x P(x)) \wedge A \equiv \forall x(P(x) \wedge A)$
   (d) $(\exists x P(x)) \wedge A \equiv \exists x(P(x) \wedge A)$

6. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

7. Show that $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x((P(x) \wedge Q(x)))$ are not logically equivalent.

8. Let P(x), Q(x), and R(x) be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and P(x),Q(x), and R(x).
   (a) All clear explanations are satisfactory.
   (b) Some excuses are unsatisfactory.
   (c) Some excuses are not clear explanations.
   (d) Does (c) follow from (a) and (b)? (*)

9. Let L(x,y) be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.
   (a) Everybody loves Jerry.
   (b) Everybody loves somebody.
   (c) There is somebody whom everybody loves.
   (d) Nobody loves everybody.
   (e) There is somebody whom Lydia does not love.
   (f) There is somebody whom no one loves.
   (g) There is exactly one person whom everybody loves.
   (h) There are exactly two people whom Lynn loves.
   (i) Everyone loves himself or herself.
   (j) There is someone who loves no one besides himself or herself.

10. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
   (a) $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$
   (b) $\forall x \forall y \exists z(z = (x + y)/2)$

11. Rewrite each of these statements so that negations appear only within predicates $\neg\exists y(\forall x\exists z T(x,y,z) \vee \exists x\forall z U(x,y,z))$

12. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x\forall y(P(x) \vee Q(y))$ where all quantifiers have the same nonempty domain, are logically equivalent. (The new variable y is used to combine the quantification correctly.)

13. Show that $\forall x P(x) \wedge \exists x Q(x)$ is logically equivalent to $\forall x\exists y(P(x) \wedge Q(y))$, where all quantifiers have the same nonempty domain.

14. Convert the following statements into First order logic?
Some children will eat any food
$C(x)$: means x is a child
$F(x)$: x is food
$Eat(x,y)$: x eats y

    (a) $\exists x\forall y(C(x) \wedge F(x,y) \wedge F(y))$
    (b) $\forall x\exists y(C(x) \wedge F(x,y) \wedge F(y))$
    (c) $\exists x[C(x) \wedge \forall y(F(y) \rightarrow Eat(x,y))]$
    (d) $\exists x[C(x) \rightarrow \forall y(F(y) \wedge Eat(x,y))]$

15. $\forall x[p(x) \rightarrow Q(x)] \wedge \forall x[p(x) \rightarrow R(x)]$ is equivalent to

    (a) $\forall x[p(x) \rightarrow Q(x) \wedge R(x)]$
    (b) $\forall x[p(x) \rightarrow Q(x) \vee R(x)]$
    (c) $\forall x[(p(x) \rightarrow Q(x)) \rightarrow R(x)]$
    (d) $\exists x[p(x) \rightarrow Q(x) \vee R(x)]$

16. "Tortoises defeats rabbits"
Convert it into first order logic?

    (a) $\forall x\forall y[tortoise(x) \wedge rabbit(y) \wedge defeat(x,y)]$
    (b) $\forall x\exists x[tortoise(x) \wedge rabbit(y) \wedge defeat(x,y)]$
    (c) $\forall x\forall y[tortoise(x) \wedge rabbit(y) \rightarrow defeat(x,y)]$
    (d) $\exists x\exists y[tortoise(x) \wedge rabbit(y) \rightarrow defeat(x,y)]$

17. Convert the following statement into first order logic. "Children do not go to school whenever they are unwell."
Here $child(x)$ means x is a child.
$Unwell(x,y)$ means x is unwell on day y
$Location(x,y,z)$ means location of x on day y is z.

    (a) $\forall x\forall y[child(x) \wedge unwell(x,y) \rightarrow location(x,y,home)]$
    (b) $\exists x\forall y[child(x) \wedge unwell(x,y) \rightarrow location(x,y,home)]$
    (c) $\forall x\forall y[child(x) \wedge location(x,y,home) \rightarrow unwell(x,y)]$

(d) $\exists x \forall y[child(x) \wedge location(x, y, home) \rightarrow \sim unwell(x, y)]$

18. Convert the following statement into first order logic statement? "All towers are of the same color"
$Tower(x)$: x is a tower.
$Color(x, y)$: x is of color y.

(a) $\forall x \exists y[Tower(x) \rightarrow Color(x, y)]$
(b) $\exists x \forall x[Color(x, y) \rightarrow Tower(x)]$
(c) $\exists x \forall y[Tower(x) \rightarrow Color(x, y)]$
(d) $\forall x \exists x[Color(x, y) \rightarrow Tower(x)]$

19. $S_1 : \forall x \exists y \forall z[x + y = z]$
$S_2 : \exists x \forall y \exists z[x + y = z]$
Where x, y, z are real numbers. Which of the following statement is true?

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) None

20. $\forall x P(x) \wedge \exists x Q(x)$ is equivalent to

(a) $\forall x \exists y[P(x) \wedge Q(y)]$
(b) $\exists x \forall y[P(x) \wedge Q(y)]$
(c) $\exists x \exists y[P(x) \wedge Q(x)]$
(d) $\forall x[P(x) \rightarrow Q(x)]$

21. Which of the following statements are correct?
$S_1 : \forall x[P(x) \vee Q(x)] \Leftrightarrow \forall x P(x) \vee \forall x Q(x)$
$S_2 : \forall x[P(x) \wedge Q(x)] \Leftrightarrow \forall x P(x) \wedge \forall x Q(x)$

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) None

22. Which of the following statements are correct?
$S_1 : \exists x[P(x) \vee Q(x)] \Leftrightarrow \exists x P(x) \vee \exists x Q(x)$
$S_2 : \exists x[P(x) \wedge Q(x)] \Leftrightarrow \exists x P(x) \wedge \exists x Q(x)$

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) None

23. Which of the following statements are correct?
$S_1 : \exists x[P(x) \rightarrow Q(x)] \Leftrightarrow \exists x P(x) \rightarrow \exists x Q(x)$
$S_2 : \exists x P(x) \rightarrow \exists x Q(x) \Leftrightarrow \exists x[P(x) \rightarrow Q(x)]$

    (a) Only S_1
    (b) Only S_2
    (c) Both S_1 and S_2
    (d) None

24. Negate the following first order logic statement?
$\forall x \exists y[P(x) \rightarrow Q(y)]$

    (a) $\exists x \forall y[P(x) \rightarrow Q(y)]$
    (b) $\exists x \forall y[P(x) \wedge \sim Q(y)]$
    (c) $\exists x \forall y[Q(x) \rightarrow P(y)]$
    (d) $\exists x \forall y[P(x) \wedge Q(y)]$

25. consider the following description
$P(x)$: x has passed physics exam
$Q(x)$: x has passed chemistry exam
"There is exactly one person who have passed both exams"

    (a) $\forall x \forall y[P(x) \wedge Q(y) \wedge x = y]$
    (b) $\forall x \forall y[P(x) \wedge Q(y) \rightarrow x = y]$
    (c) $\forall x \forall y[x = y \rightarrow P(x) \wedge Q(y)]$
    (d) $\forall x \forall y[x \neq y \rightarrow \sim P(x) \wedge \sim Q(y)]$

26. Which of the following first order logic statements are equivalent?

    (a) $\sim \forall x\{P(x) \vee \exists y[Q(y) \wedge P(y)]\} \equiv \exists x\{\sim P(x) \wedge \forall y[(P(y) \rightarrow \sim Q(y)) \vee (Q(y) \rightarrow \sim P(y))]\}$
    (b) $\sim \forall x\{P(x) \vee \exists y[Q(y) \wedge P(y)]\} \equiv \exists x\{\sim P(x) \wedge \forall y[\sim (P(y) \rightarrow \sim Q(y)) \vee (\sim Q(y) \rightarrow \sim P(y))]\}$
    (c) $\sim \forall x\{P(x) \vee \exists y[Q(y) \wedge P(y)]\} \equiv \exists x\{\sim P(x) \wedge \forall y[(P(y) \rightarrow \sim Q(y)) \vee (\sim Q(y) \rightarrow \sim P(y))]\}$
    (d) $\sim \forall x\{P(x) \vee \exists y[Q(y) \wedge P(y)]\} \equiv \exists x\{\sim P(x) \wedge \forall y[\sim (P(y) \rightarrow \sim Q(y)) \vee (Q(y) \rightarrow \sim P(y))]\}$

27. Which of the following formulas is a formalization of the sentence:
"There is a computer which is not used by any student"

    (a) $\exists x(Computer(x) \wedge \forall y(\neg Student(y) \wedge \neg Uses(y, x)))$
    (b) $\exists x(Computer(x) \rightarrow \forall y(Student(y) \rightarrow \neg Uses(y, x)))$
    (c) $\exists x(Computer(x) \wedge \forall y(Student(y) \rightarrow \neg Uses(y, x)))$

(d) $\exists x(Computer(x) \rightarrow \forall y(\neg Student(y) \wedge \neg Uses(y,x)))$

28. There is a student who is loved by every other student.

    (a) $\exists x(Student(x) \wedge \forall y(Student(y) \wedge \neg(x=y) \rightarrow Loves(y,x)))$
    (b) $\exists x(Student(x) \rightarrow \forall y(Student(y) \wedge \neg(x=y) \wedge Loves(y,x)))$
    (c) $\exists x(Student(x) \wedge \forall y(Student(y) \wedge \neg(x=y) \vee Loves(y,x)))$
    (d) $\exists x(Student(x) \wedge \forall y((Student(y) \wedge \neg(x=y)) \rightarrow Loves(y,x))$

29. "Everyone has exactly one best friend"
    Which of the following first order logic statements correctly represents above English statement? $BF(x,y) = x$ and y are best friends
    $S_1 : \forall x \exists y \forall z(BF(x,y) \wedge \sim BF(x,z) \rightarrow (y \neq z))$
    $S_2 : \forall x \exists y(BF(x,y) \rightarrow \forall z[(y \neq z)] \rightarrow \sim BF(x,z))$

    (a) Only S_1
    (b) Only S_2
    (c) Both S_1 and S_2
    (d) None

30. Suppose that $P(x,y)$ means "x is a parent of y" and $M(x)$ means "x is a male".
    If $F(v,w)$ equals $M(v) \wedge \exists x \exists y(P(x,y) \wedge P(x,v) \wedge (y \neq v) \wedge P(y,w))$, the meaning of the expression $F(v,w)$ is

    (a) v is a brother of w
    (b) v is an uncle of w
    (c) v is a grandfather of w
    (d) v is a nephew of w