

Chapter 4

Counting

1. There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
2. A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student can leave answers blank?
3. How many strings are there of four lowercase letters that have the letter x in them?
4. How many 5-element DNA sequences (a) contain only A and T? (b) do not contain C?
5. How many positive integers between 1000 and 9999 inclusive are not divisible by either 5 or 7? How many strings of three decimal digits a) do not contain the same digit three times?
6. How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?
7. How many ways are there to seat four of a group of ten people around a circular table where two seating are considered the same when everyone has the same immediate left and immediate right neighbor?
8. How many ways are there to seat six people around a circular table where two seating are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?
9. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if a) the bride must be next to the groom? b) the bride is not next to the groom? c) the bride is positioned somewhere to the left of the groom?

10. How many bit strings of length eight contain either three consecutive 0s or four consecutive 1s? The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? (Note that the name of a variable may contain fewer than eight characters.)
11. Use mathematical induction to prove the product rule for m tasks from the product rule for two tasks.
12. How many bit strings of length 10 contain (a) at least four 1s? (b) an equal number of 0s and 1s?
13. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]
14. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
15. A circular r -permutation of n people is a seating of r of these n people around a circular table, where sittings are considered to be the same if they can be obtained from each other by rotating the table. Find a formula for the number of circular r -permutations of n people.
16. How many ways are there for a horse race with four horses to finish if ties are possible?
17. In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed?
18. How many ways are there to assign three jobs to five employees if each employee can be given more than one job?
19. How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?
20. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose a) a dozen bagels with at least one of each kind? b) a dozen bagels with at least three egg bagels and no more than two salty bagels?
21. How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it? How many ways are there to distribute 12 distinguishable objects into six distinguishable boxes so that two objects are placed in each box?

22. How many positive integers less than 1,000,000 have exactly one digit equal to 9 and have a sum of digits equal to 13?
23. There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?
24. How many different strings can be made from the letters in AARDVARK, using all the letters, if all three As must be consecutive
25. How many ways are there to travel in xyz space from the origin $(0, 0, 0)$ to the point $(4, 3, 5)$ by taking steps one unit in the positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative x, y, or z direction is prohibited, so that no backtracking is allowed.)
26. In bridge, the 52 cards of a standard deck are dealt to four players. How many different ways are there to deal bridge hands to four players?
27. In how many ways can a dozen books be placed on four distinguishable shelves if no two books are the same, and the positions of the books on the shelves matter? [Hint: Break this into 12 tasks, placing each book separately. Start with the sequence 1, 2, 3, 4 to represent the shelves. Represent the books by $b_i, i = 1, 2, \dots, 12$. Place b_1 to the right of one of the terms in 1, 2, 3, 4. Then successively place b_2, b_3, \dots , and b_{12}]