COMBINATORICS

Basic Rules:

Sum Rule: Let S = S_1 U S_2……S_n, where S_1, S_2… S_n are pair wise disjoint sets then
|S|=|S_1|+|S_2|+………..|S_n|

Example: Number of cards which are either face cards or 10 number cards?
Solution: Here S = S_1U S_2, where S_1=face cards, S_2=10 number cards.
S_1, S_2 sets are mutually disjoint sets.
Hence |S|=|S_1|+|S_2|
|S|=4*3+4
=16

Note: Face cards are cards of type J, Q, K.

Product Rule:

T is a task and it can be done with sequence of sub tasks T_1, T_2… T_n
$T=T_1\to T_2\to T_3\to T_4\ldots\ldots\to T_n$.
The number of ways T can be done
i.e. |T|=|T_1|*|T_2|*|T_3|*|T_4|…………..|T_n|.
Where T_1, T_2, T_3………are independent tasks.

Example: The number of binary sequence of length n?
Solution: How to form a binary sequence of length n?
It can be done by performing a task sequence called $T_1\to T_2\ldots T_n$ where each T_i is filling i^{th} position in the sequence.
Since filling each position is independent of others
|T|=|T_1|*|T_2|………*|T_n|
|T_1|=no of ways of filling 1^{st} position it can be done in 2 ways.
Similarly each T_i can be done in 2 ways.
|T|=2*2*………..*n times=2^n

Indirect Counting: Sometimes it will be very difficult to design a task T=T1→T2…→Tn where each task is independent, that type of tasks can be done in indirect way.

Example: The numbers between 0-9999 which contain at least one 5?
Solution:
Wrong way of solution: Any number between 0-9999 can be viewed with 4 - different positions.

$\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}$

Select a position and place 5.after that fill any number in the remaining three positions.

T = T5→T_ _ _

Where T5 = selecting one position and filling 5
T_ _ _ = Filling any number in remaining three positions.
T5, T_ _ _ are not independent tasks here. Why?
While doing task T5 you are selecting a position, in the remaining

Positions you can fill any number so filling three fives also possible.

Hence a number $\begin{pmatrix} \sim & 5 & 5 & 5 \\ \underline{5} & \underline{\sim} & \underline{\sim} & \underline{\sim} \\ 1 & 2 & 3 & 4 \end{pmatrix}$

5 5 5 5 is counted here.

But if we select 2^{nd} position and fill 5, then in the remaining positions if we fill 5,5,5 then 5 5 5 5 will be formed again.

Then we can say, for every doing of T_5 we cannot do T_ _ _ in same number of ways. We say that there is a dependency between tasks.

Hence we go for indirect way.

The numbers with atleast one 5 = total numbers – numbers without 5

=10*10*10*10 - 9*9*9*9

Total numbers can be formed by filling each position by any of 10 digits.

The numbers without 5 can be formed by filling each position by any 9 options.

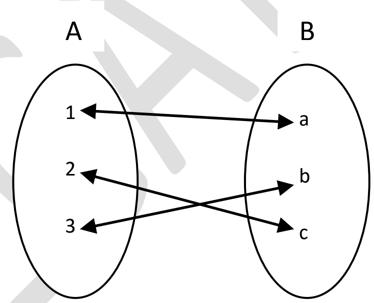
Here answer=10^4-9^4

One-One correspondence:

It is more powerful technique generally used in counting.

S1, S2 are two sets, where |S2| is known and |S1|is unknown or |S2| is unknown and |S1|is known. if it is known that for every element in set S1 there is exactly one element in S2 and for every element in S2 there is one element in S1.

Hence we can decide the size of S1 which is equal to |S2|. See the picture



Here we can observe that whenever there is a one to one correspondence between A and B then there would be equal number of elements in both sets.

The analysis of 4^{th} template:

<n-diff objects, r-combinations, each object can be repeated>

The number of such r-combinations= ?

<a1, a2, a3…an> r-combinations

Here are some such combinations

r a_1's, 0 a_2's 0 a_3's…… 0 a_n's

r-1 a_1's 1 a_2's 0 a_3's…… 0 a_n's

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r-2 a_1's 1 a_2's 1 a_3's…… 0 a_n's

There is a similar problem to above problem which has same no of solutions.
i.e. $x_1 + x_2 + x_3 + \ldots x_n = r$
Where each $x_i >= 0$
Example:
r + 0 + 0 + 0 +…………...+ 0 = r
r-1 + 1 + 0 + 0 +………..+ 0 = r
r-2 + 1 + 1 + 0 +………..+ 0 = r
The solutions of this problem have a correspondence with solutions of combination problem.
Hence both solutions are equal.
What is one – one correspondence?
We have two sets s1 and s2 and if for every element of s1 can be mapped to exactly one element of s2 and if every element of s2 can be mapped to exactly one element of s2 then s1 and s2 are said to be in one-one correspondence.
Similarly solutions of x1 + x2 +…… + xn = r , where each xi ≥ 0 have one- one correspondence with balls distribution problem.
Balls distribution Problem:

□ + □ + □ □ = n
1 2 3 r

We have n-different boxes and r balls to be distributed among them, then the number of such distributions has one-one correspondence with equation problem.
X1 + x2 +…… + xn = r, where each xi ≥ 0.

□ + □ + □ …… □ = r balls ↔ x1 + x2 + …… xn = r
. 1 2 3 n

[r balls] [0 balls] …… [0 balls] ↔ r+0+0…….+0 = r

[R-1 balls] [1 ball] [0 balls] ↔ r-1 + 1 + 0 = r

Hence the no of solutions are same in 2cases. Now consider the box distribution solution and corresponding binary sequences.
Example: We have 3 balls and 3 boxes then each box distribution has a correspondence with binary sequence of 2 1's and 3 0's(How ?).

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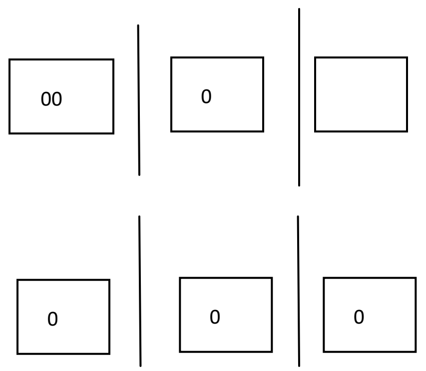


$\leftrightarrow$ 00|0| $\leftrightarrow$ 00101

$\leftrightarrow$ 0|0|0

We can easily find that the number of binary sequences with n-1 1's and r 0's has correspondence to Box distribution with r balls.
The no of binary sequences with n-1 1's, r 0's?

__ __ __ __ __ __ __
1 2 3 n+r-1

$T = T_{\text{place 1's}} \rightarrow T_{\text{place zero's}}$
For every distribution of placing of one's, placing of zero's is independent.
$|T| = |T_{\text{place1's}}| * |T_{\text{place 0's}}|$
The no of total positions= n+r-1 and one's are n-1.
$|T_{\text{place 1's}}| = {}^{(n+r-1)}C_{n-1}$ [it is a selection problem]
$|T_{\text{place 0's}}|$ = only one way [we are doing it after $|T_{\text{place 1's}}|$]
$|T| = {}^{(n+r-1)}C_{n-1} = {}^{(n+r-1)}C_r$ [since ${}^nC_r = {}^nC_{n-r}$]

5th template

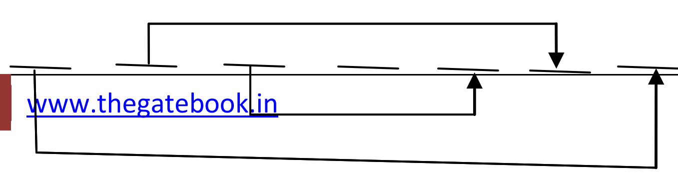
<n objects <a_1, t_1 times>, <a_2, t_2 times>…..<a_k, t_k times>>
Where $t1+t_2+\ldots\ldots+t_k= n$
The no of arrangements for such n objects = $\frac{n!}{t1!\,t2!\,t3!\ldots tk!}$

Example:
<5a's, 3b's, 3c's> the no of arrangements of 10 objects?
$= \frac{10!}{5!3!2!}$

Practice Questions with Solutions:
1) The number of palindromes with exactly 7 positions? (Using English alphabet)
a) 26^7 b) 26^4 c) 26^3 d) 26^5
Solution: Each palindrome can be viewed with 7 positions.

___ ___ ___ ___ ___ ___ ___

Forming a palindrome is nothing but filling 7 positions with appropriate alphabets
$T=T_1\rightarrow T_2\rightarrow T_3\rightarrow T_4\rightarrow T_5\rightarrow T_6\rightarrow T_7$



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Filling position 7 cannot be done in 26 independent ways but can be done in one way. since to form a palindrome 7th position must be equal to 1st position.
similarly T5 and T6 also.
$|T|=|T_1| * |T_2| * |T_3| * |T_4| * |T_5| * |T_6| * |T_7|$
Where $|T_5|=|T_6|=|T_7|=1$ way
|T|=26*26*26*26*1*1*1
$=26^4$

2) .The number of integers between 0-9999 which contains exactly two 4's ?
Solution: wrong method
T= T4→T4→T_ _
Where T4=filling 4 in some position.
|T|= |T4|*|T4|*|T_ _|
=4*3*9*9
=12*81
=972
The problem with this solution is first 2 tasks are not independent.
T4=fixing 4 in some position.

4 _ _ _ , _ 4 _ _ , _ _ 4 _ , _ _ _ 4
One way second way third way fourth way

2nd task |T4|= fixing 4 in some position.
After doing task one, doing task 2 can be done in 3 ways but they are not independent.
(Think about it?)
Hence we go for other task sequence that is $T=T_{\text{fix two4's}} \rightarrow T_{\text{fix other positions}}$
$T_{\text{fixes two 4's}}$ = _ _ _ _
1 2 3 4
To fix the two 4's, the positions can be taken from any of the 4 positions. Order of the position is not important .hence it is a combination of 2 out of 4.
Hence 4C2 ways it can be done.
$T_{\text{fix other}}$ = _ _
Fixing 2 other positions with any other number without using 4.
Hence $|T_{\text{fix other two}}|$= 9*9
$T_{\text{fix two 4's}}$ and $T_{\text{fix other two}}$ are independent tasks

4 _ 4 _   4 4 _ _
1 2 3 4   1 2 3 4
↑ ↑   ↑ ↑

After fixing 4's into two positions fixing other two positions is always independent.
$|T|=|T_{\text{fix two 4's}}|*|T_{\text{fix other 2 positions}}|$
$= {}^4C_2 * 9^2$

3) The number of 4 digit even numbers with all distinct digits?
a) 2296 b) 2620 c) 4536 d) 2240

Solution: wrong method:

$\overline{T_1}$ $\overline{T_2}$ $\overline{T_3}$ $\overline{T_4}$

If we perform $T1 \to T_2 \to T_3 \to T_4$

T_1=filling the first position =$|T_1|$=9 ways (0 is not allowed)

T_2=filling the 2nd position

$|T_2|$=9 ways (0 is allowed and any number is allowed other than number used in T_1)

$|T_3|$=8 ways

T4=filling even number in the 4th position

$|T_4|$= 5 (wrong!) (since we do not know how many even numbers have been used in the previous fillings)

$|T_5|$ cannot be uniquely decided (do not know whether it is 2 or 3 or 4 or 5)

Right method:

→ split the problem into 2 cases (since '0' causes the problem)

S_1= all the even no's ending with 0'

S_2= all the even no's ending with other than zero

$S = S_1 \cup S_2$ = all the even no's

$|S|=|S_1|+|S_2|$

S_1 can be formed by doing below task.

$T = T_0 \to T_1 \to T_2 \to T_3$

T_0 = fixing 0 in the last position

T_1 = fixing some no other than zero in 1st position

T_2 = fixing some no in 2^{nd} position

T_3 = fixing some no in 3^{rd} position

$|T|=|T_1|*|T_2|*|T_3|*|T_0|$

Ex: 2 3 4 1

$\underline{\mathbf{9\ ways}}$ $\underline{\mathbf{8ways}}$ $\underline{\mathbf{7ways}}$ $\underline{\mathbf{1way}}$

X X X 0

$|T|$ = 9 * 8 * 7 * 1

1 →2→3→4 is the order of filling.

Similarly S2 can be formed by doing following task sequence

$T = T_0 \to T_1 \to T_2 \to T_3$

$\underline{\mathbf{8ways}}$ $\underline{\mathbf{8ways}}$ $\underline{\mathbf{7ways}}$ $\underline{\mathbf{4ways}}$

T_1 T_2 T_3 T_0

$|T|$=8*8*7*4

$S = S_1 \cup S_2 = |S_1|+|S_2| = |S|$

= 9 * 8 * 7 + 8 * 8 * 7 * 4

$|S|$= 2296

4) The no of subsets of a set of n elements?

a) 2^n b) n^2 c) n! d) n (n+1)/2

Solution: If there is a one-one correspondence between two sets S1&S2 then we say $|S1|=|S2|$

S1= the no of subsets of set of n elements

S2= the no of binary sequences of length n
Example:
Subset {e1, e2, e3} corresponds to binary sequence
1 1 1 0 0 ……… **0**.
e_1 e_2 e_3 e_4 e_5 e_n
Similarly ϕ set has correspondence with 00000…..0 so on
The no of subsets = the no of binary sequences
The no of binary sequences $=2^n$
Hence the no of subsets = 2^n

5) How many ways 10 teachers and 15 students committee can be formed from 15 teachers and 20 students so that teacher A refuses to serve when student B is in the committee?
Solution: The solutions can be divided into 2 cases
Case1: The committees without A
Case2: The committees without A and without B
Case1:

Teachers	Students
15	20

We need 10 teachers, 15 students
$T = T_{\text{select 10 teachers}} \rightarrow T_{\text{select 15 students}}$
Since A should not be included in selection of teachers
So $|T_{\text{select 10 teachers}}| = {}^{(15-1)}C_{10}$
$|T_{\text{select 15 students}}| = {}^{20}C_{15}$
$|T| = {}^{14}C_{10} * {}^{20}C_{15}$
Case 2:
$T = T_{\text{select 10 teachers with including A}} \rightarrow T_{\text{select 15 students-B}}$
To select 10 teachers where A is must included, we have to select only 9 teachers from remaining 14 teachers and add teachers A. So that it would be a 10 teacher committee which includes teacher A, hence it is ${}^{14}C_9$ ways
$T_{\text{select 15 students}}$ = select 15 students from 20 students except B
$= {}^{(20-1)}C_{15} = {}^{19}C_{15}$ ways
$|T| = {}^{14}C_9 * {}^{19}C_{15}$ ways

6) How many ways letters of MISSISSIPPI can be arranged?
Solution: <n objects, <a1, t1> <a2, t1> <a2, t2>…….<ak, tk> >
<11 objects, <M, I> <I, 4><P, 2><S, 4> >
The no of arrangements = $\frac{n!}{t1!t2!\ldots\ldots tk!} = \frac{11!}{4!\,2!\,4!\,1!}$

2nd method:

__ __ __ __ __ __ __ __ __ __ __
1 2 3 4 5 6 7 8 9 10 11

Forming an arrangement of 11 letters can be done by following task sequence
$|T_{\text{filling M 's}}|$=r's can be done by following task sequence

$T_{\text{filling M's}} \rightarrow T_{\text{filling I's}} \rightarrow T_{\text{filling S's}} \rightarrow T_{\text{filling P's}}$

$T_{\text{filling M's}}$ = we have 11 positions and we have to select 1 position to place M

= ${}^{11}C_1$ ways it can be done

$|T_{\text{filling M's}}| = {}^{11}c_1$

$|T_{\text{filling I's}}|$ = we have 10 remaining positions and need to place 4 I's. It is again combination problem

$= {}^{10}C_4$ ways it can be done

Similarly $T_{\text{filling S's}}$ can be done in ${}^{6}C_4$ ways

$T_{\text{filling P's}}$ can be done in ${}^{2}C_2$ ways

$|T| = {}^{11}C_1 * {}^{10}C_4 * {}^{6}C_4 * {}^{2}C_2$

$= \frac{11!}{4!4!2!1!}$

7) How many binary sequences of 2n length have exactly n 0's & n 1's?

Solution: We can follow this task sequence.

$T = T_{\text{fill 0's}} \rightarrow T_{\text{fill 1's}}$

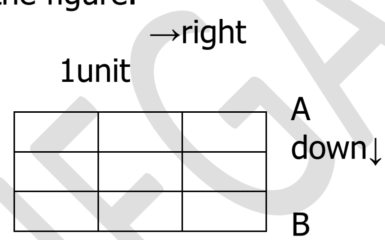
$T_{\text{fill 0's}}$ = Filling n 0's in 2n positions since 0's cannot be arranged themselves, only selection of n positions is important, hence ${}^{2n}C_n$ ways it can be done.

$T_{\text{fill 0's}}$= since the remaining positions are n and the 0's to be placed are n.

So only one way they can be arranged in the remaining places.

Hence = $T = T_{1's} \rightarrow T_{0's} = |T_{1's}| * |T_{0's}| = {}^{2n}C_n * 1 = {}^{2n}C_n$

8) Consider the figure.



The person at point A is allowed to move one unit down or one unit right. The no of ways person can be reached to point B from point A?

To reach from A to B person has to take 3 rights and 3 ups in some order.

Example: RRRUUU, RURURU, RRUURU…..

Each different path equal to arranging 3R's, 3U's in a line

Hence it can be done in = $\frac{6!}{3!\,3!}$ ways.

9) The number of non negative integral solutions for $x_1 + x_2 = 20$ where each $x_i \geq 2$?

Solution: We do not have direct equation

We have formula for equation like $y_1 + y_2 = r$

Where each $y_i \geq 0$

That's why we will put one-one correspondence between solutions of our problem with solutions of $y_1 + y_2 = r$

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Where each $y_1 \geq 0$

$x_1 + x_2 = 20$ | take $y_1 = x_1 - 2$

$x_1 \geq 2 \;\; x_2 \geq 0$ | $y_2 = x_2 - 2$

then $y_1 + y_2 = x_1 + x_2 - 4$

$y_1 + y_2 = 20 = 4 = 16$

where $y_1 = x_1 - 2$, $y_2 = x_2 - 2$

since $x_1 \geq 2$ and $x_2 \geq 2$ then $y_1 \geq 0$, $y_2 \geq 0$

8+12=20 $\leftrightarrow$ 6+10=16

13+7+20 $\leftrightarrow$ 11+5=16

.

.

Hence the no of solutions are same in 2 cases.

We have a formula for 2nd case

Where $y_1 + y_2 = 16$

Each $y_i \geq 0$

${}^{(n+r-1)}C_r$ where n=2 r=16

${}^{(16+2-1)}C_{16} = {}^{17}C_{16} = {}^{17}c_1 = 17$

10) The no of binary sequences on k 1's and n 0's so that no consecutive 1's are present in the sequence?

Solution: Place n 0's in the row

↑ 0 ↑ 0 ↑ 0 ↑ 0 ↑ 0 ↑ 0……… ↑ 0 ↑

It can be done in 1 way.

After that we get n+ 1 positions where each position can be filled with 1. The number of 1's = k and no of positions = n+1. The no of ways of placing 1's in n+1 positions $= {}^{(n+1)}C_k$

Principles of Mutual Exclusion and Inclusion:

Consider a set $S = S_1 \cup S_2$

$|S|=|S_1|+|S_2|-|S_1 \cap S_2|$

To count set S above principle can be used .which is called principle of mutual inclusion and exclusion.

Similarly if $S = S_1 \cup S_2 \cup S_3$ then

|S| can be calculated with below formula

$|S|=|S_1|+ |S_2| + |S_3| - |S_1 \cap S_2| - |S_1 \cap S_3| - |S_2 \cap S_3| + |S_1 \cap S_2 \cap S_3|$

Similarly above formula can be extended for $S = S_1 \cup S_2 \cup S_3 \ldots\ldots .S_n$

$|S| = \sum_{i=1}^{n} |si| - \sum_{i,j \in 1}^{n} |si \cap sj| + \ldots\ldots\ldots (-1)^{n-1} |S_1 \cap S_2 \cap \ldots\ldots S_n|$

Example: The number of binary sequences of length 5 whose first bits is 0 or last bit is 1?

Such sequences can be denoted as $S= S_1 \cup S_2$

Where S_1 = binary sequences starts with 0

S_2 = binary sequences end with 1

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$|S| = |S_1 \cup S_2|$

$|S_1|$= sequence of length 5 starts with 0

$|S_2|$= sequence of length 5 ends with 1

$|S| = |S_1| + |S_2| - |S_1 \cap S_2|$

$|S_1|$ = sequence of length 5 starts with 0

_0_ __ __ __ __

To form a binary sequence of length 5 starts with 0 the first position should be zero and other 4 positions can be anything (either 0 or 1).

Each position can be filled in 2 ways. There are 4 such positions. Hence 2^4 ways such sequences can be formed.

Similarly to form a binary sequence of length 5 ending with 1, the last position should be 1.

__ __ __ __ _1_

Remaining 4 positions can be filled 24 ways.

$|S_1 \cap S_2|$=all the binary sequences starting with zero and ending with 1.

0 __ __ __ _1

Remaining 3 positions can be filled in 2^3 ways

Hence $|S_1 U S_2| = |S_1|+|S_2|-|S_1 \cap S_2|$

$= 2^4+2^4-2^3$

$= 32-8 = 24$

Example 2: The numbers between 1-1000 which are divisible by either 4 or 6?

Solution: S_4 U S_6 is the required set.

S_4 = all the no's divisible by 4

S_6 = all the no's divisible by 6

$|S_4 U S_6| = |S_4|+|S_6|-|S_4 U S_6|$

S_4 = all the no's divisible by 4 = $\left\lfloor \frac{1000}{4} \right\rfloor$ = 250

S_6 = all the no's divisible by 6 = $\left\lfloor \frac{1000}{6} \right\rfloor$ = 166

$|S_4 U S_6|$ = all the no's divisible by 4 and 6 that means they are divisible by LCM (4, 6)

Note: A number divisible by 4, 6 if and only if number divisible by LCM (4, 6)

$|S_4 U S_6| = \left\lfloor \frac{1000}{LCM\ (4,6)} \right\rfloor = \left\lfloor \frac{1000}{12} \right\rfloor = 83$

$|S_4 U S_6|$=|S4| + |S6| - |S4 ∩ S6|

= 250 + 166 - 83

= 416 - 83

=333

Example 3: The no of relatively prime numbers of 30?

Relatively prime no:

X is called relatively prime to y iff X ≤ Y

no common prime factors exist between x & y

Prime factors of 30 = 2, 3, 5

A number is relatively prime no to 30 then it cannot be divisible by 2 and 3 and 5

S_2 = all the no's divisible by 2
S_3 = all the no's divisible by 3
S_5 = all the no's divisible by 5
$\overline{S2}\cap\overline{S3}\cap\overline{S5}$ set denotes the numbers which are not divisible by 2 and 3 and 5
We don't have formula for $|\overline{S2} \cap \overline{S3} \cap \overline{S5}|$

Hence we go for (opposite) complement set.
i.e. All the no's which are divisible by 2 and 3 and 5
i.e. $\overline{S_2US_3US_5} = \overline{S2} \cap \overline{S3} \cap \overline{S5}$
$|\overline{S2} \cap \overline{S3} \cap \overline{S5}|$ = total no's between 1 to 30 - $\overline{S_2US_3US_5}$ (all numbers divisible by 2 or 3 or 5)
$S_2US_3US_5$ = $|S_2| + |S_3| + |S_5| - |S_2\cap S_3| - |S_3\cap S_5| - |S_2\cap S_5| + |S_2\cap S_3\cap S_5|$

$$= \left\lfloor\frac{30}{2}\right\rfloor + \left\lfloor\frac{30}{3}\right\rfloor + \left\lfloor\frac{30}{5}\right\rfloor - \left\lfloor\frac{30}{\text{LCM }(2,3)}\right\rfloor - \left\lfloor\frac{30}{\text{LCM }(3,5)}\right\rfloor - \left\lfloor\frac{30}{\text{LCM }(2,5)}\right\rfloor + \left\lfloor\frac{30}{\text{LCM }(2,3,5)}\right\rfloor$$

=15+10+6-5-2-3+1
= 31 – 10 + 1
= 22

The number of relatively prime numbers = total – numbers which are not relatively primes
Numbers which are not relatively prime = numbers which are divisible by 2 or 3 or 5
= | S2 ∪ S3 ∪ S5 | = 22
Therefore, the number of relatively primes = 30 – 22 = 8.

Example 4: The number of derangements for 5 elements?
Solution: Derangements of n-objects.
An arrangement is called derangement of n objects if no element is in the previous position.

Example:	1	2	3	4	5
Derangement:	5	4	2	3	1

For 1, 2, 3, 4, 5 ⇒ 5, 4, 3, 2, 1 is called derangement.
Consider an arrangement which is not a derangement
1 2 3 4 5
1 3 2 5 4 → not a derangement
It is not a derangement because 1 is in its correct position. Think about the non-derangements. In each non-derangement 1 or 2 or 3 or 4 or 5 would be in correct position. So we calculate non-derangements first then after that derangements can be calculated easily
Number of non – derangements: $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$
Where S_1= set of non – derangements because 1 is in its correct position.
S_2 = set of non – derangements because 2 is in its correct position.
S_3 = set of non – derangements because 3 is in its correct position.

S_4 = set of non – derangements because 4 is in its correct position.

S_5 = set of non – derangements because 5 is in its correct position

$|S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5| = \sum_{i=1}^{5} |Si| - \sum_{i,j \in 1}^{5} |Si \cap Sj| + \ldots\ldots (-1)^{5-1} | S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 |$

$|S_1|$ = number of non – derangements where 1 is in its correct position

1 _ _ _ _

Arrangement of other 4 objects = 4! Ways

Therefore, $|S_1| = 4!$, $|S_2| = 4!$, $|S_3| = 4!$, $|S_4| = 4!$, $|S_5| = 4!$

$|S_1 \cap S_2|$ = the number of non-derangements where 1 and 2 are in correct position.

2 _ _ _ _

Therefore, $|S_1 \cap S_2| = 3!$, $|S_2 \cap S_3| = 3!$, $|S_3 \cap S_4| = 3!$, $|S_4 \cap S_5| = 3!$

$|S_1 \cap S_2 \cap S_3|$ = The number of non-derangements where 1 and 2 and 3 are in its correct position.

= 1 2 3 _ _

Therefore, $|S_1 \cap S_2 \cap S_3| = 2!$, $|S_2 \cap S_3 \cap S_4| = 2!$, $|S_3 \cap S_4 \cap S_5| = 2!$

Similarly $|S_1 \cap S_2 \cap S_3 \cap S_4| = 1!$

$|S_1 \cap S_2 \cap S_3 \cap S_4 \cap S_5| = 0!$

$|S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5| = \sum_{i=1}^{5} |Si| - \sum_{i,j \in 1}^{5} |Si \cap Sj| + \ldots\ldots (-1)^{5-1} |S_1 \cap S_2 \cap S_3 \cap S_4 \cap S_5|$

The number of sets of type $S_i = {}^5C_1$

The number of sets of type $S_i \cap S_j = {}^5C_2$

The number of sets of type $S_i \cap S_j \cap S_k = {}^5C_3$

$\sum |Si| = {}^5C_1 * 4!$

$\sum |S_i \cap S_j| = {}^5C_2 * 3!$

Hence $|S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5| = {}^5C_1 * 4! - {}^5C_2 * 3! + {}^5C_3 * 2! - {}^5C_4 * 1! + {}^5C_5 * 0!$

$= 120 - 60 + 20 - 5 + 1 = 76$

The number of derangements = Total arrangements – total non-derangements

Total arrangements $= {}^5P_5 = 5! = 120$

Number of derangements = 120 – 76 = 44.